Institute of Education

Mathematics Education in the Digital Age: progress, reality and directions for future research

Prof Dame Celia Hoyles
UCL Institute of Education
University College London
U.K.

Keynote at launch of Asian Centre for Maths Education (ACME),
Shanghai Oct 2018

谢谢范教授和他所有棒棒的学生，
谢谢华东师范大学的邀请。

structure of my talk

1. Reflections on my keynote at ICME, Mexico 2008

Transforming the mathematical practices of learners and teachers through digital technology

Research in Mathematics Education, Published online: 04 Jul 2018

With response by Paul Drijvers *Freudenthal Institute, Utrecht University*

collaborative research & mutual exchanges

2008 ICME talk: enduring premises……

- centrality of **representation**

Abstractions are *shaped by* and *expressed in* the medium (Noss & Hoyles, Windows on Mathematical Meanings, 1996)

  - semiotic mediation

- crucial importance of fostering **mathematical**

epistemic agency …the group assumes for the ownership of ideas (Bereiter, 2002; Scardamalia, 2002)

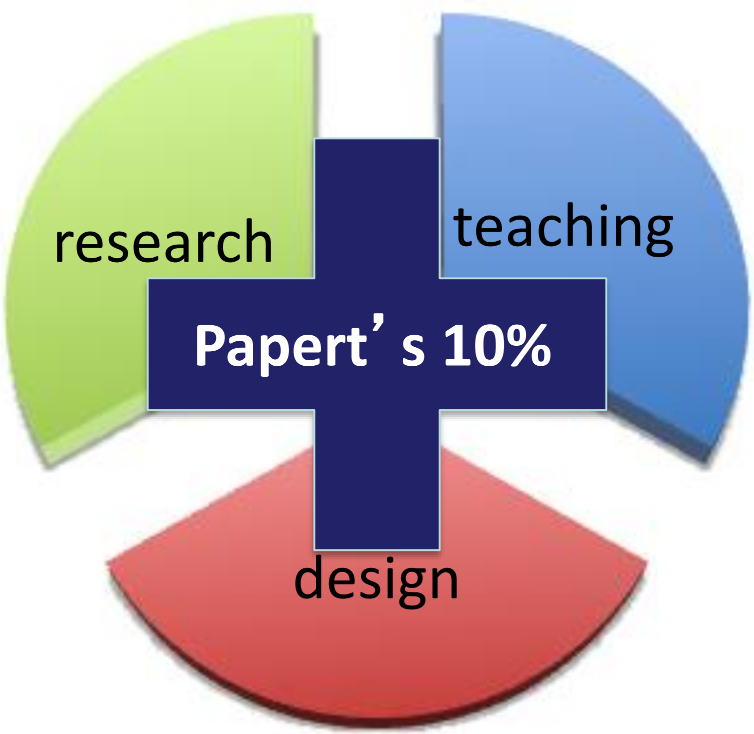
so

less 'guess what is in the mind of the teacher' (the didactical paradox, Brousseau)

  - discuss, share, challenge & reflect

- foster **epistemic agency**

Exploiting digital technologies for 'better' mathematics learning requires ....



ICMI Study: Hoyles. C and Lagrange J-B (eds) (2010) *Mathematics Education and Technology- Rethinking the terrain* Springer

the potential of digital technology ICME 2008

dynamic & visual 2D & 3D tools to explore in **shared** space	changing how mathematics is taught & learned
tools to outsource processing power	changing the collective focus of attention
new representational infrastructures for maths	changing what can be learned and by whom
connections between school and learners' culture	bridging the gap: school maths and problem solving in the 'real world: **joint exploration of authentic problems**
connectivity	opening new opportunities for shared knowledge construction & student autonomy: **genuinely collaborative**
intelligent support for the teacher	making **exploratory** learning environments more possible

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connections between school and learners' culture

bridging the gap: school maths and problem solving in the 'real world: **joint exploration of authentic problems**

Aydin, H. & Monaghan, J. **Encouraging students' problem posing through importing visual images into mathematical software** *Teaching Mathematics and its Applications: An International Journal of the IMA*, Volume 37, Issue 3, 5 September 2018

the potential of digital technology ICME 2008

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connectivity

opening new opportunities for shared knowledge construction & student autonomy: **genuinely collaborative**

Huge literature on collaborative learning, group work, pair work, peer tutoring

Face-to-face and online

need to design to catalyse sharing of .....

- resources
- information
- student solutions or part-solutions

... generate **mathematical** discussion

- **process** by which knowledge is constructed
- **justifications & refutations**

the potential of digital technology ICME 2008

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intelligent support for the teacher

making exploratory learning environments more possible

- harness **Artificial Intelligence** techniques in the interests of mathematical learning

‘user’ modelling based on mathematics education research

learning trajectories more explicit

intelligent support for mathematical generalisation

Hoyles, Noss, Mavrikis, Geraniou & computer science team
2007- 2011

the potential of digital technology ICME 2008

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tools to outsource processing power

changing the collective focus of attention

Example

easy and free graph-drawing software means students can use quick (and internet browser friendly) **graph plotting** for checking and challenging incorrect thinking

does losing the skill of *graph sketching* matter?

the potential of digital technology ICME 2008

dynamic & visual **2D & 3D** tools to explore in **shared** space

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new representational infrastructures for maths

changing what can be learned and by whom

...the difficulty of a mathematical idea often derives from the system with which it is expressed

—imagine just how difficult it would be to remember the various procedural rules of calculus (like the chain rule) without Leibniz's elegant notation. (Kaput, Hoyles & Noss, 2002)

Coding: an international phenomenon

Eric Schmidt
Chief Executive of Google
visited England 2011

"I was flabbergasted to learn that today Computer Science isn't even taught as standard in UK schools"

"Your IT curriculum focuses on teaching how to use software, ***but gives no insight into how it's made".***

Everybody needs to program 2012


theguardian | TheObserver

News | Sport | Comment | Culture | Business | Money | Life & style

News > Education > Computer science and IT

Why all our kids should be taught how

John Naughton

The Observer, Saturday 31 March 2012 20.15 BST

Jump to comments (288)

Bring coding into school maths

ScratchMaths project ☺

New Computing National Curriculum in England <u>2014</u>

The ScratchMaths project

design research to develop a 2-year curriculum for 9-11 year olds in England

- aligned to the national computing *and* national

Computational thinking

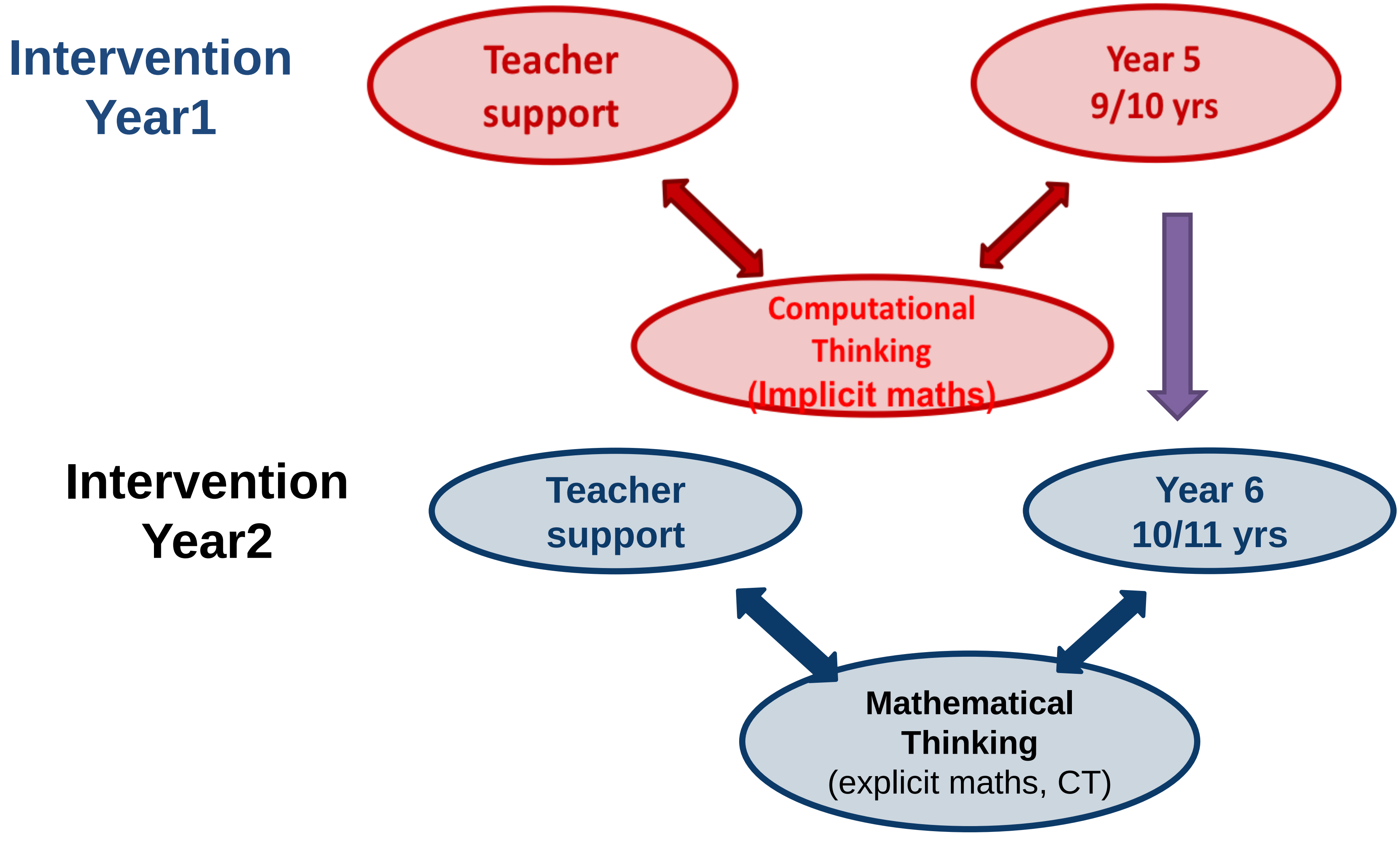
and

Mathematical thinking

Method: iterative design and trialling in four 'design schools'

Glimpse of the classroom research case studies

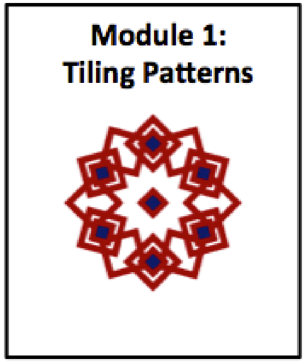
Two-year “instructional sequence”

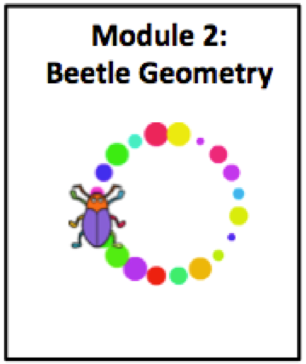


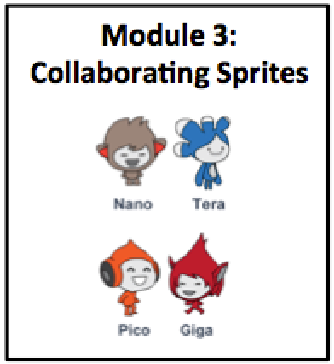
Knowledge Lab

UCL

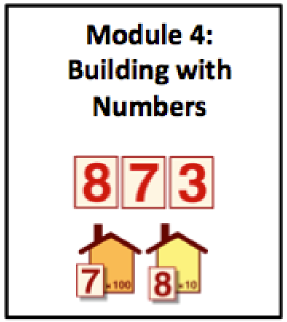
Year 5 (9-10 yrs) – Computing focus (20+ hours of teaching materials)

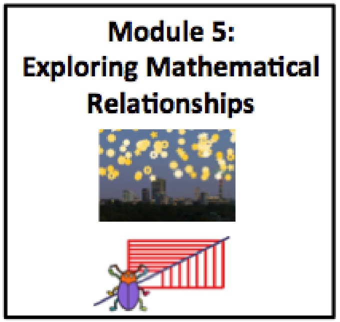


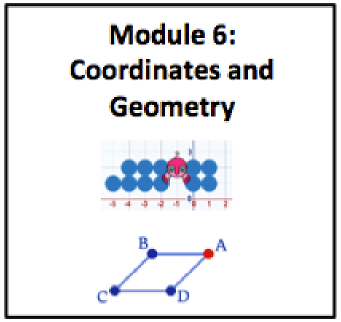




Year 6 (10-11 yrs) – Mathematics focus (20+ hours of teaching materials)





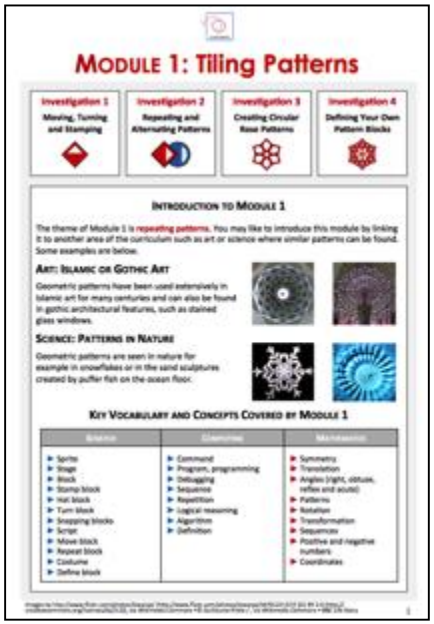


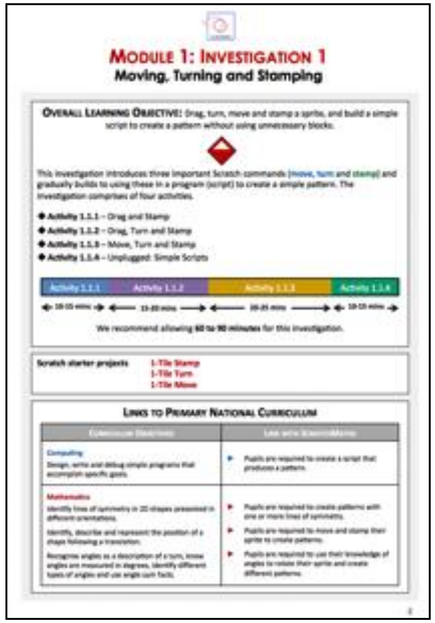
Freely available through UCL website http://www.ucl.ac.uk/scratchmaths

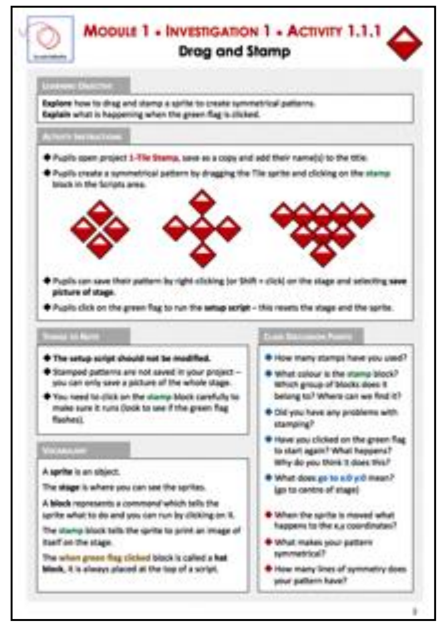
Knowledge Lab

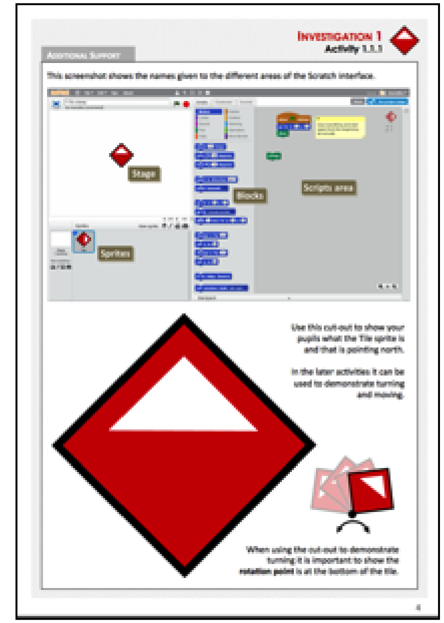
UCL

Extensive Teacher Support Materials and Example Scripts

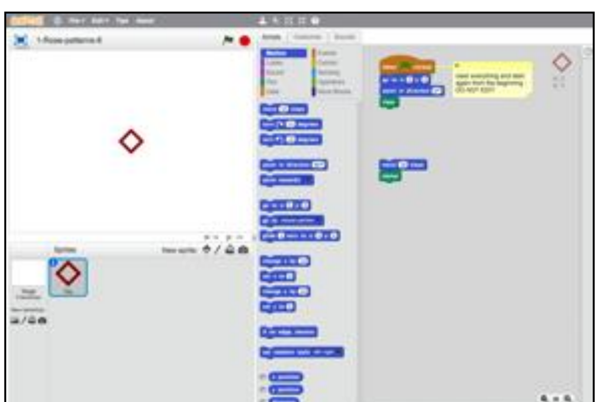


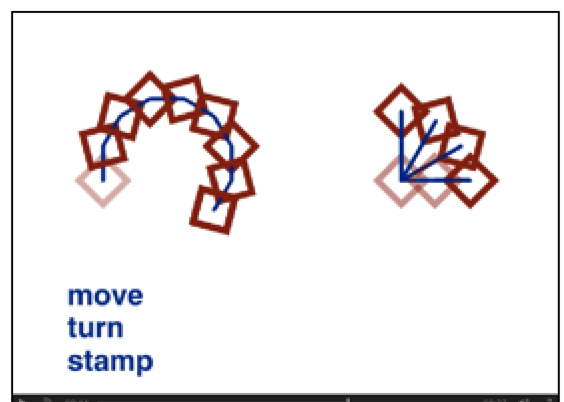






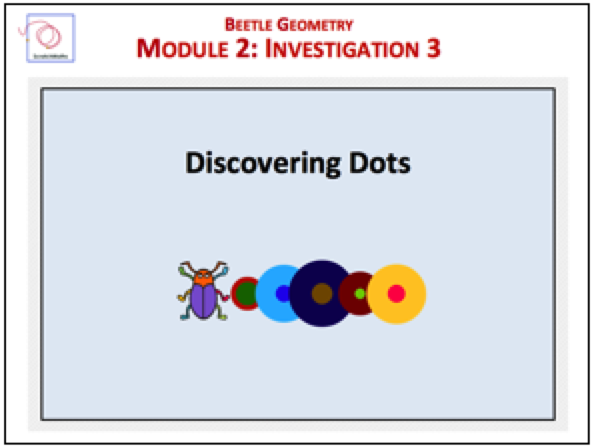
Scratch Starter Projects

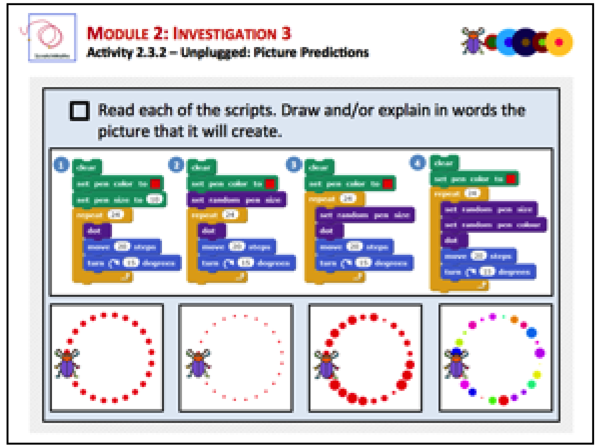


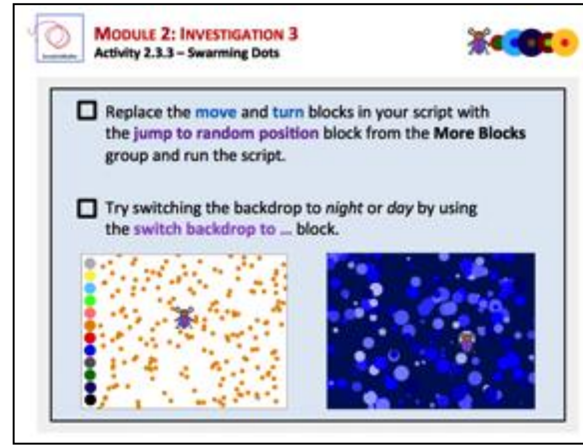


Support Videos

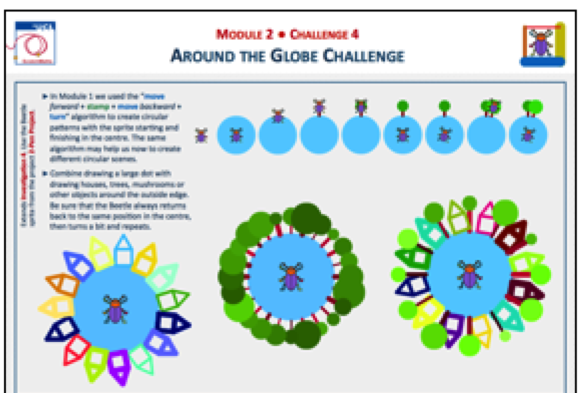
Presentation Slides for every lesson







Additional challenges, vocabulary and reference posters



Pedagogical framework (5Es)

Explore: Investigate, try things out yourself, debug in reaction to feedback

Envisage: Have a goal in mind, predict outcome of program *before trying it*

Explain: Explain what you have done, articulate reasons behind your approach to yourself & others

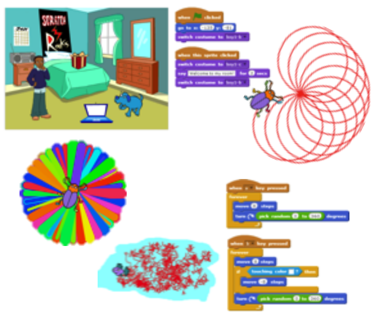
Exchange: Collaborate & share, try to see a problem from another's perspective as well as defend your own approach and compare with others.

bridgE: Make **explicit** links to the mathematics curriculum

Glimpse of the classroom research case studies

Quantitative results in Jan 2019

Overview of SM research process



Initial Design → **Design School testing** → **Professional Development** → **Treatment schools (55 schools)**

Initial Design
- Theory and literature
- Design workshops
- School observations

Design School testing
- Observed intervention 4 schools
- Discussions with pupils and teachers

Professional Development
- Two days of professional development
- Discussions with teachers

Treatment schools (55 schools)
- Pre/post surveys
- Observed same activity in a subset of schools
- Interviewed teachers and pupils

The ScratchMaths project…

National trial

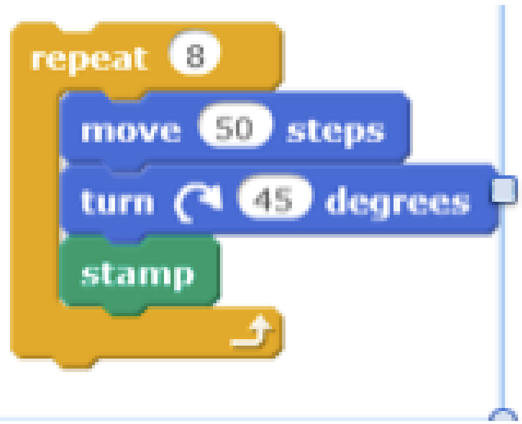
•110 English primary schools recruited in early 2015, 2,986 students

Control schools put on 'wait list' to receive intervention one year after treatment schools

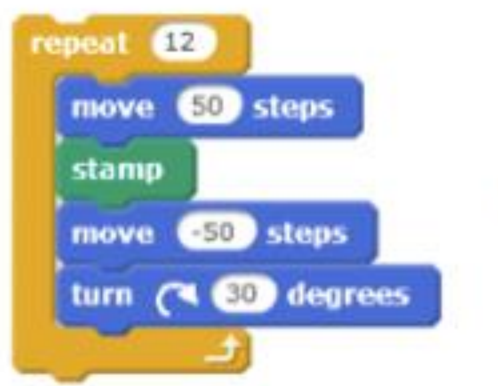
prior attainment as measured by national standardised mathematics assessment at age 8 years

example: Compare two algorithms

Algorithm 1

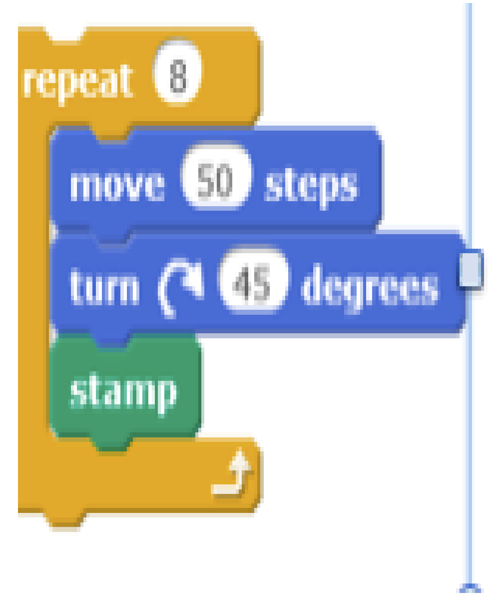


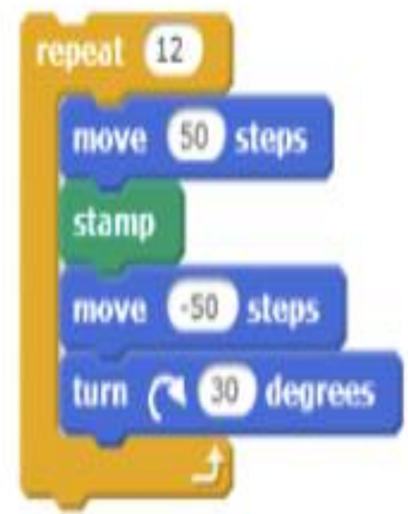
Algorithm 2



two learning goals

- notion of algorithm
- 360 as total turn


repeat 8
move 50 steps
turn 45 degrees
stamp


repeat 12
move 50 steps
stamp
move -50 steps
turn 30 degrees

Unplugged tasks….

Which algorithm was used to create each of the drawings?

Algorithm 1

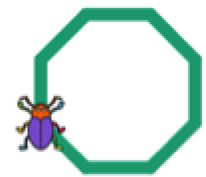
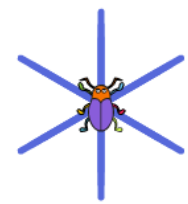
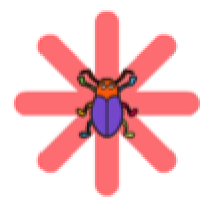
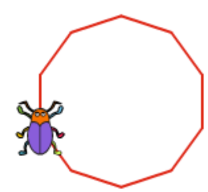
Repeat the following:

- Move
- Turn

Algorithm 2

Repeat the following:

- Move
- Move backwards
- Turn

Drawing	Which algorithm does it use?
1 	
2 	
3 	
4 	

Module 2: Investigation 3

Activity 2.3.2 – Unplugged: Picture Predictions

☐ Read each of the scripts. Draw and/or explain in words the picture that it will create.

1
```
clear
set pen color to [red]
set pen size to 10
repeat 24
    dot
    move 20 steps
    turn ↻ 15 degrees
```

2
```
clear
set pen color to [red]
set random pen size
repeat 24
    dot
    move 20 steps
    turn ↻ 15 degrees
```

3
```
clear
set pen color to [red]
repeat 24
    set random pen size
    dot
    move 20 steps
    turn ↻ 15 degrees
```

4
```
clear
set pen color to [red]
repeat 24
    set random pen size
    set random pen colour
    dot
    move 20 steps
    turn ↻ 15 degrees
```

?	?	?	?

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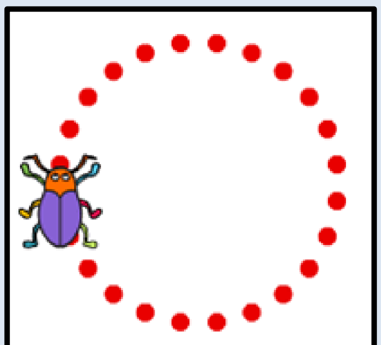
repeat 24
    dot
    move 20 steps
    turn ↻ 15 degrees
```

3
```
clear
set pen color to [red]
repeat 24
    set random pen size
    dot
    move 20 steps
    turn ↻ 15 degrees
```

4
```
clear
set pen color to [red]
repeat 24
    set random pen size
    set random pen colour
    dot
    move 20 steps
    turn ↻ 15 degrees
```



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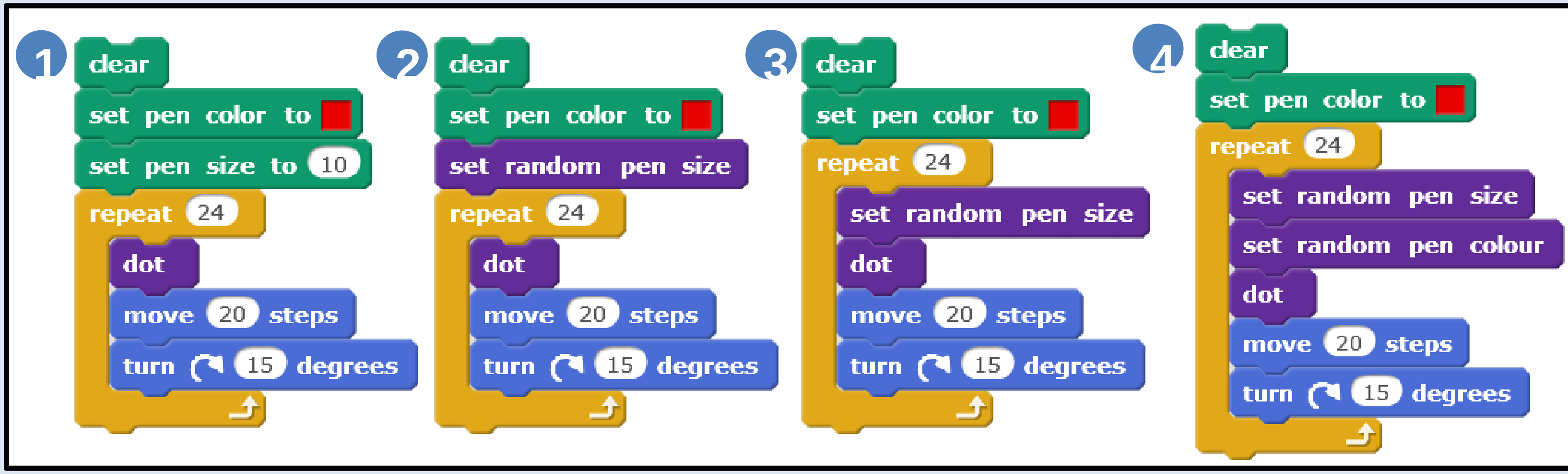

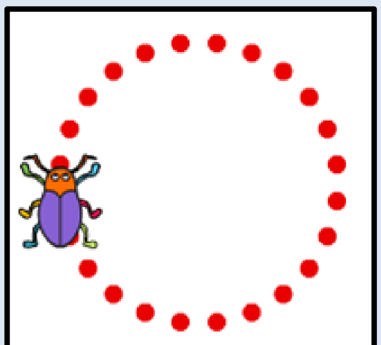

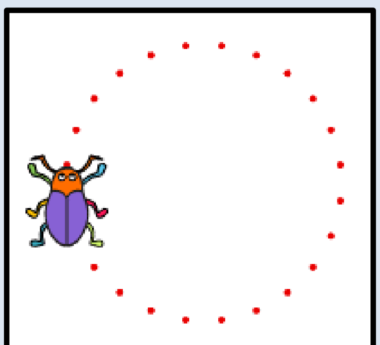

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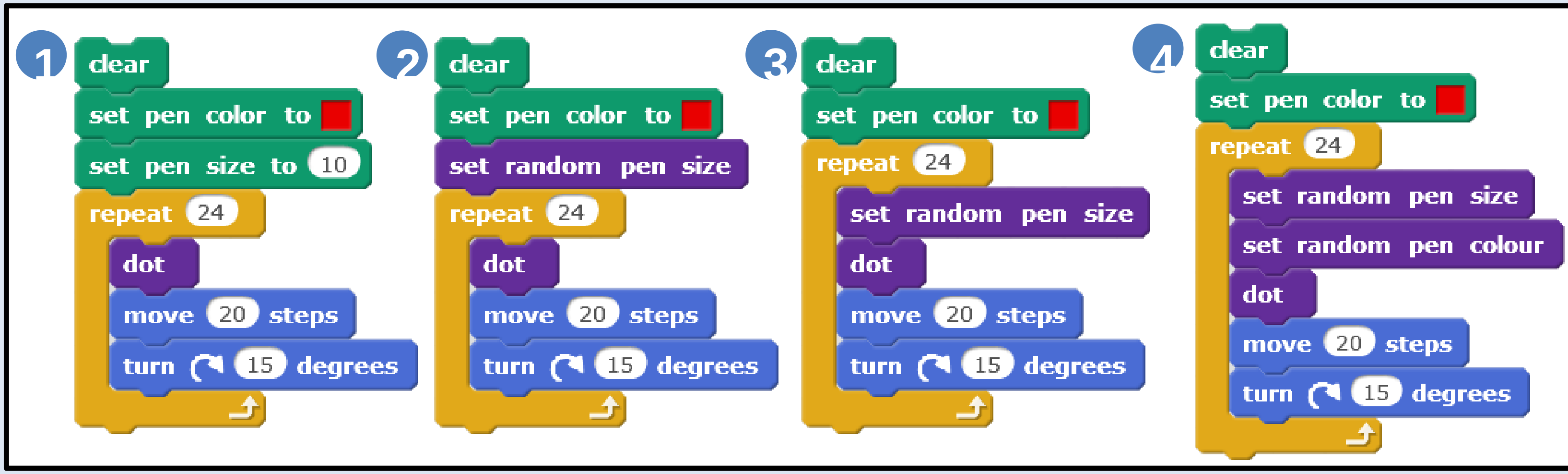

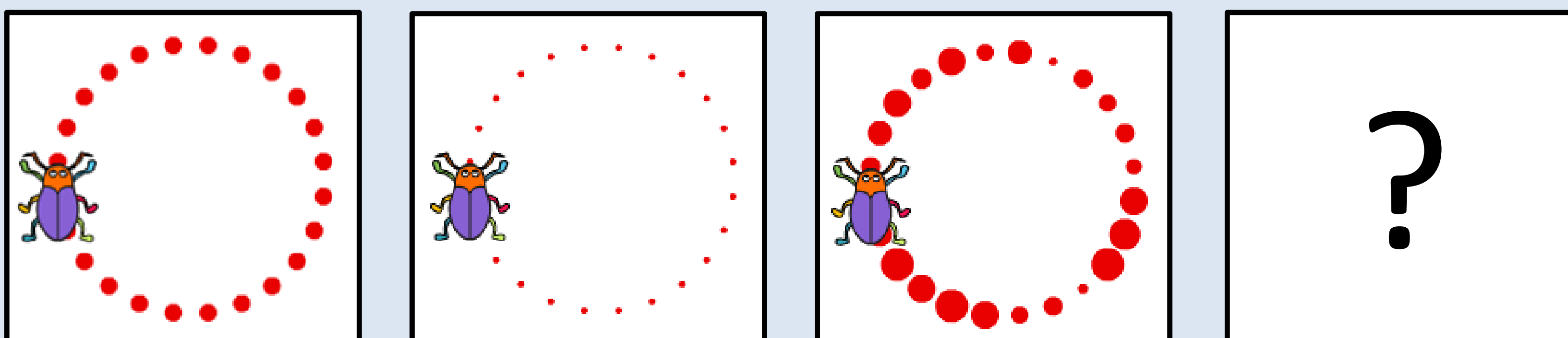

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```

2
```
clear
set pen color to ■
set random pen size
repeat 24
    dot
    move 20 steps
    turn ↻ 15 degrees
```

3
```
clear
set pen color to ■
repeat 24
    set random pen size
    dot
    move 20 steps
    turn ↻ 15 degrees
```

4
```
clear
set pen color to ■
repeat 24
    set random pen size
    set random pen colour
    dot
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```
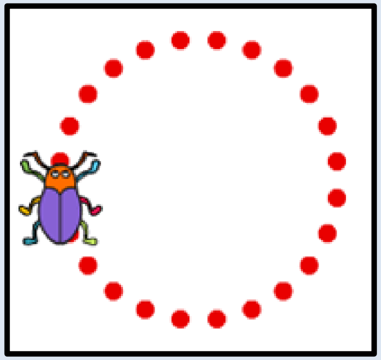
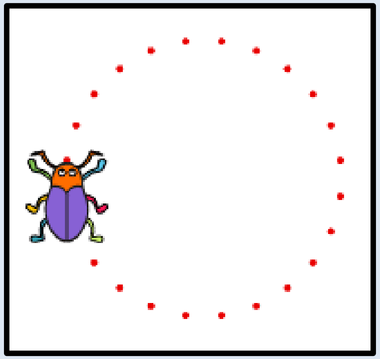
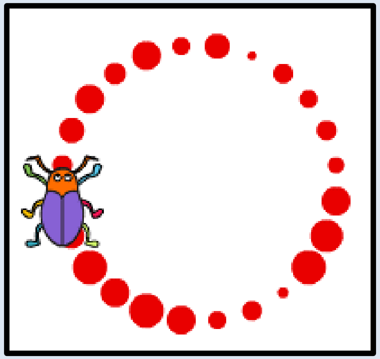
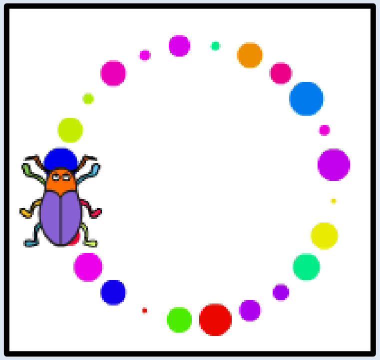
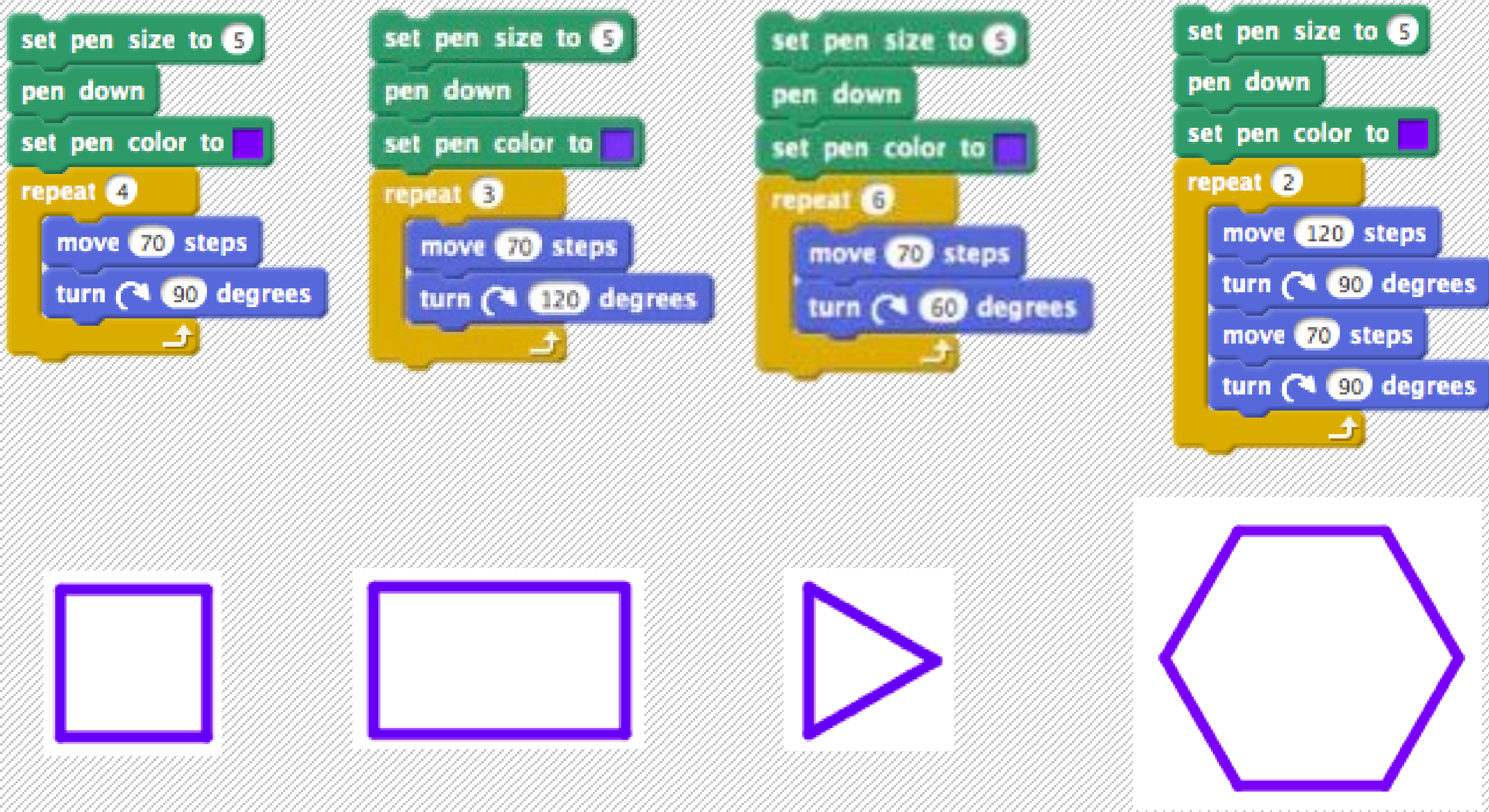









Match the script to the polygon it would draw.



Explain how you worked out your answer…

PROJECTS….

Polygon Fireworks

fruitful areas of SM research

- Understanding of algorithm (pupil and teacher)
- Affordances of computer programming for mathematics learning
- Evolution of teacher knowledge through engaging with SM
- Pupil and teacher creativity
- … **and many more**
    - engagement of girls
    - nature of teacher **professional development**
    - assessment & evaluation
    - teacher knowledge & confidence
    - teacher beliefs
    - mathematics anxiety of teachers and students…

- The gap between curriculum design & implementation: idea of **fidelity**

Fidelity: how far is an innovation implemented according to its aims and objectives?

SM derived 5 criteria as proxy measures of fidelity

1. technology access
2. curriculum progression
3. curriculum coverage
4. teaching time allocated
5. engagement in professional development (PD)

Measures 3,4,5 differentiated *high*, *medium* and *low* fidelity

Some findings…from survey data

- fidelity very high in first year but dropped dramatically in second year..

Why?

- negative impact of the high-stakes mathematics testing in mathematics at the end of school year
- teachers more able to adapt their teaching to E's in the context of teaching a 'new' subject, **computing,** while struggled to change their practice in established and higher-stakes subject, mathematics
- and they lacked lacked confidence in maths & computing

Scratchmaths in other countries

Some findings…from survey data

Lethal mutations ☹

W

Quantitative results of randomized control trial in Jan 2019

•

•

while struggled to change their practice in established and higher-stakes subject, mathematics, in which many also lacked confidence

Scratchmaths in other countries

Some findings…from survey data

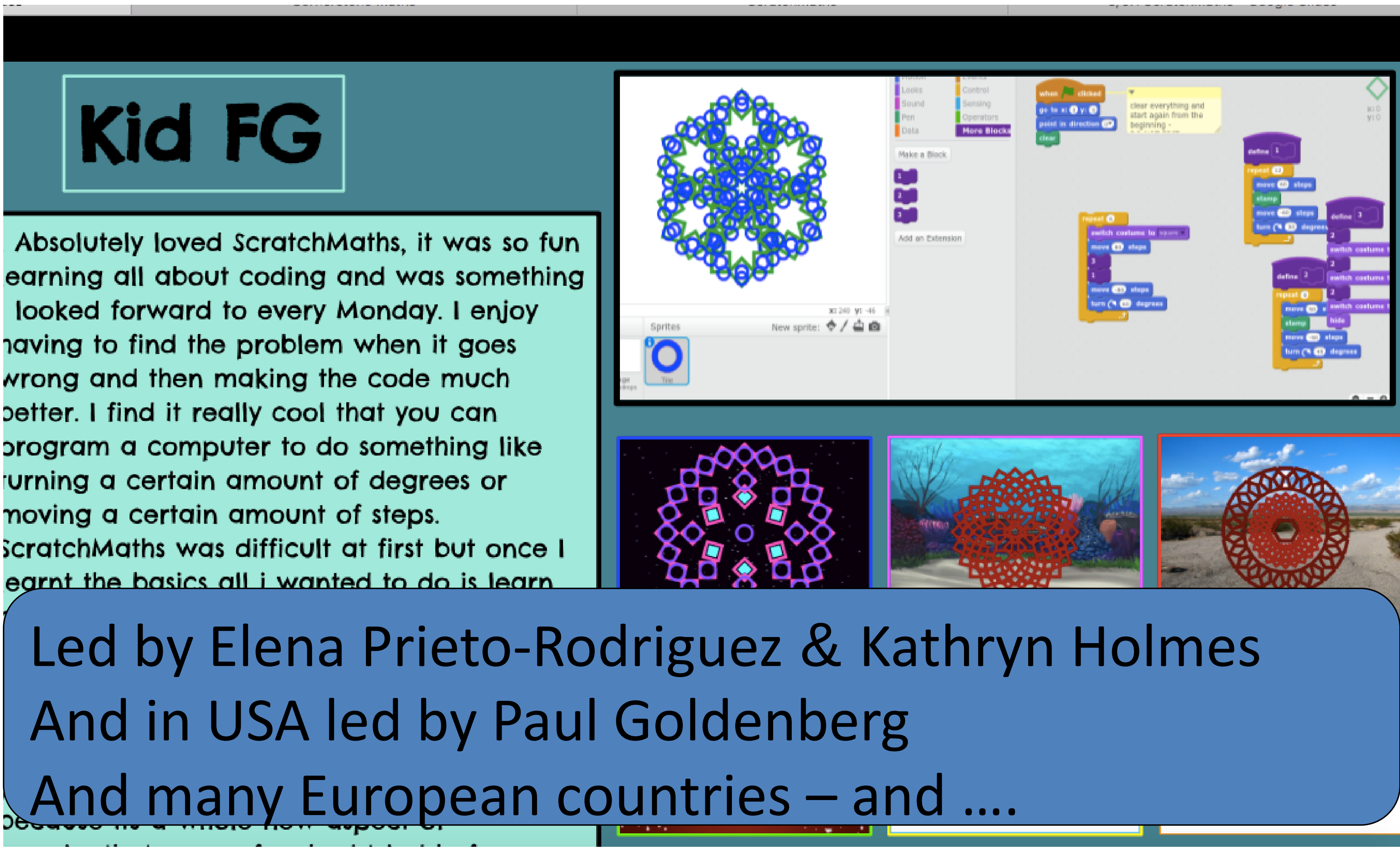
- fidelity very high in first year but dropped dramatically in second ...

Why?

- negative impact of high-stakes testing in mathematics at the end of school year
- teachers felt more able to adapt their teaching to E's in the context of teaching a 'new' subject, **computing,** while struggled to change their practice in established and high-stakes subject, mathematics, …in which many also lacked confidence

Scratchmaths in other countries

Scratchmaths in Australia



Led by Elena Prieto-Rodriguez & Kathryn Holmes

And in USA led by Paul Goldenberg

And many European countries – and ....

Scratchmaths in China

- Wei Ting from Nanjing Xiaozhuang University. run summer schools using the ScratchMaths materials

我们继7月11号之后，又迎来新的一期期待已久scratch的教学喽！！

7月18日，小先生们又来到金陵图书馆，给小朋友们进行第二次授课，在简单的排列图案中，再一次和小

我们的小朋友们在课上都十分认真并积极地举手，想回答老师的问题哦。

现场同学们的积极参与如同炎炎夏日一样热情炽烈，给我们在场的小先生和志愿者们以最大的欣慰与

足！

首先呢，小先生罗沁媛先简单在计算机上画了两个图案，一个是直线，一个是圆形。

Scratchmaths in China ctd

Dr. Hongliang Ma, Professor in Educational Technology, Shaanxi Normal University

- designed ScratchMaths learning materials aligned to the Chinese Math Curriculum Standard for grade four students
- is supervising experimental research in a primary school based on the 5Es' pedagogy of ScratchMaths.

What are differences & similarities in implementation & outcomes?

the potential of digital technology ICME 2008

dynamic & visual **2D & 3D** tools to explore in **shared** space

changing how mathematics is taught & learned

tools to outsource processing power

changing the collective focus of attention

new representational infrastructures for maths & interfaces **(touch, feel …)**

changing what can be learned and by whom

connections between school and learners' culture

bridging the gap: school maths and problem solving in the 'real world: **joint exploration of authentic problems**

connectivity

opening new opportunities for shared knowledge construction & student autonomy**: genuinely collaborative**

intelligent support for the teacher

making **exploratory** learning environments more possible

dynamic & visual **2D & 3D** tools to explore in **shared** space

changing how mathematics is taught & learned

Shift from tasks to research-informed curriculum innovation

Example: Cornerstone Maths

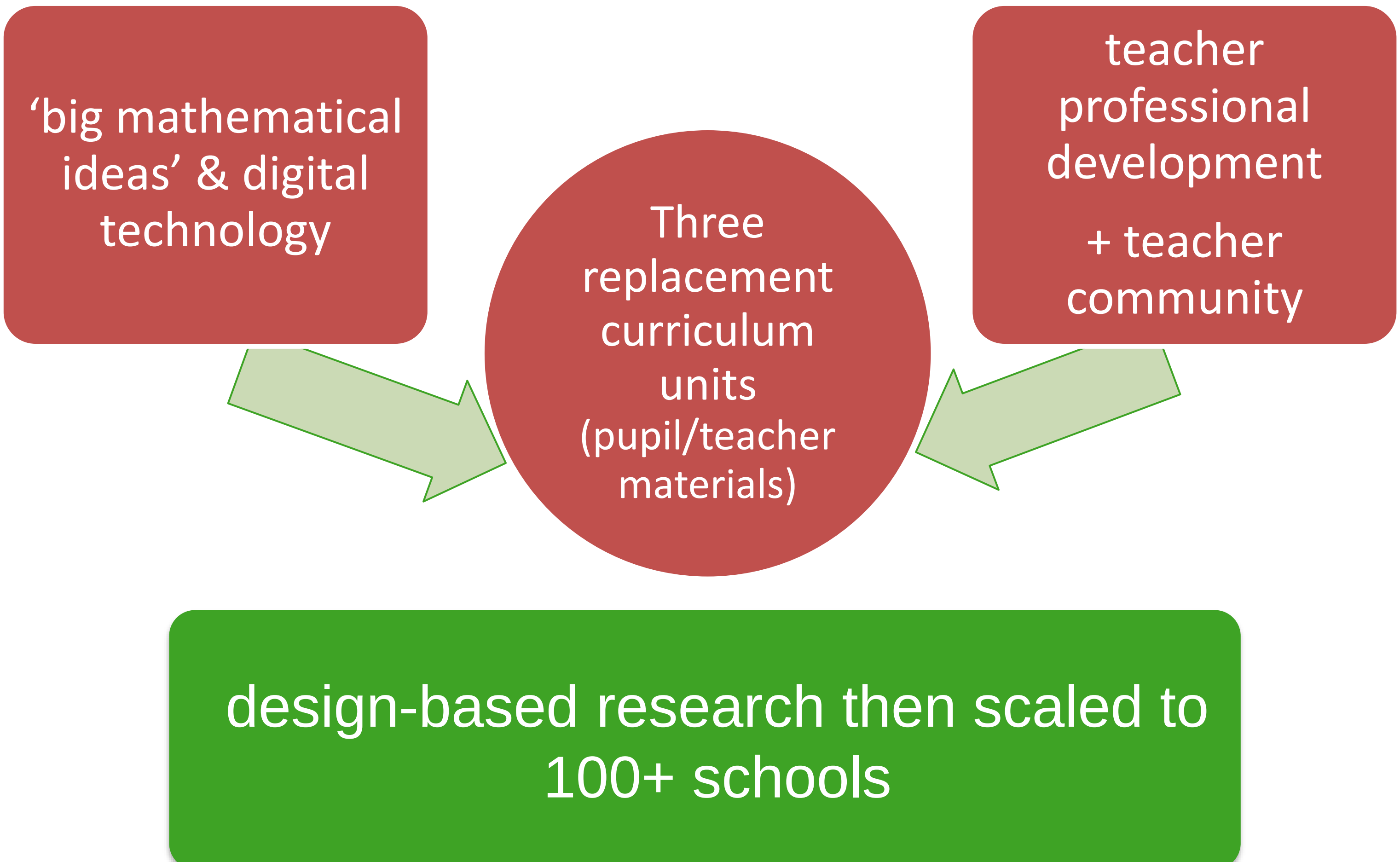
Cornerstone mathematics

Celia Hoyles, Richard Noss, Alison Clark-Wilson & SRI International (Roschelle and Vahey)

exploits the dynamic and visual nature of digital technology for students aged 11-14 years to stimulate engagement with mathematical ways of thinking by

- focusing on ‘big mathematical ideas’ that are hard to teach
- making dynamic links between key representations
- providing an environment for students to explore & solve problems within guided structured activities

Cornerstone Maths: a curriculum innovation approach



The Cornerstone Maths units

Unit 1 – linear functions

Unit 2 – geometric similarity

Unit 3 – patterns and expressions

Lethal mutations

Brown, A. L., & Campione, J. C. (1996). *Psychological theory and the design of innovative learning environments: On procedures, principles, and systems*: Lawrence Erlbaum Associates, Inc.

Landmark activity

- Use of digital technology leads to
    - cognitive breakdown
    - ‘situation of non-obviousness’
    - ‘aha’ moment: surprise, rethinking
- suggests growing appreciation of underlying concept(s)

maybe not as anticipated

Designing Mobile Games

A module on
linear functions

Context: a company creating video games for mobile phones

“You are helping this company to design new games”

Jim Kaput, SimCalc

Jeremy Roschelle & SRI collaboration

Landmark

A glimpse of Cornerstone in action

constant in the function

Potential research areas….

Instrumentation (fluency with digital tools)

Prof SC Kong Director of the Center for Learning, Teaching and Technology and Prof of Math and IT Hong Kong Education University

Organised a Summer camp around using Cornerstone

What are factors fostering 'spread'

Cornerstone is used within USA and many European countries and…..

China

Potential for cross–country research?

More cycles of design research using ‘new’ tools

- Augmented reality
- Multi-touch
- Mobile devices with touch screens,
- 3D visualisations
- ….

Ref: Recent research on geometry education: ICME-13 survey team report Nathalie Sinclair, Maria G. Bartolini Bussi, Michael de Villiers, Keith Jones, Ulrich Kortenkamp, Allen Leung, Kay Owens

dynamic & visual **2D & 3D** tools to explore in **shared** space

changing how mathematics is taught & learned

A Confession about Personalized Learning

Larry Berger, CEO of Amplify

With thanks to Paulo Blickstein

In summary..

- Digital tools are creeping into schools at varying rates…
  - which classes? age and 'ability"? presentation? flipped classrooms etc. Is tool-use transformational?

Hoyles, C., (1993) Microworlds/Schoolworlds: The transformation of an innovation. In Keitel, C., Ruthven, K. (eds) *Learning from Computers: Mathematics Education and Technology*. NATO ASI, Series F: Computer and Systems Sciences, 121, 1-17.

- Semiotic mediation
- Networking theories
- Embodiment; gesture; tool use as perceptuo-motor fluency

What new insights do the use of such theories bring?

Challenges to SM in England….

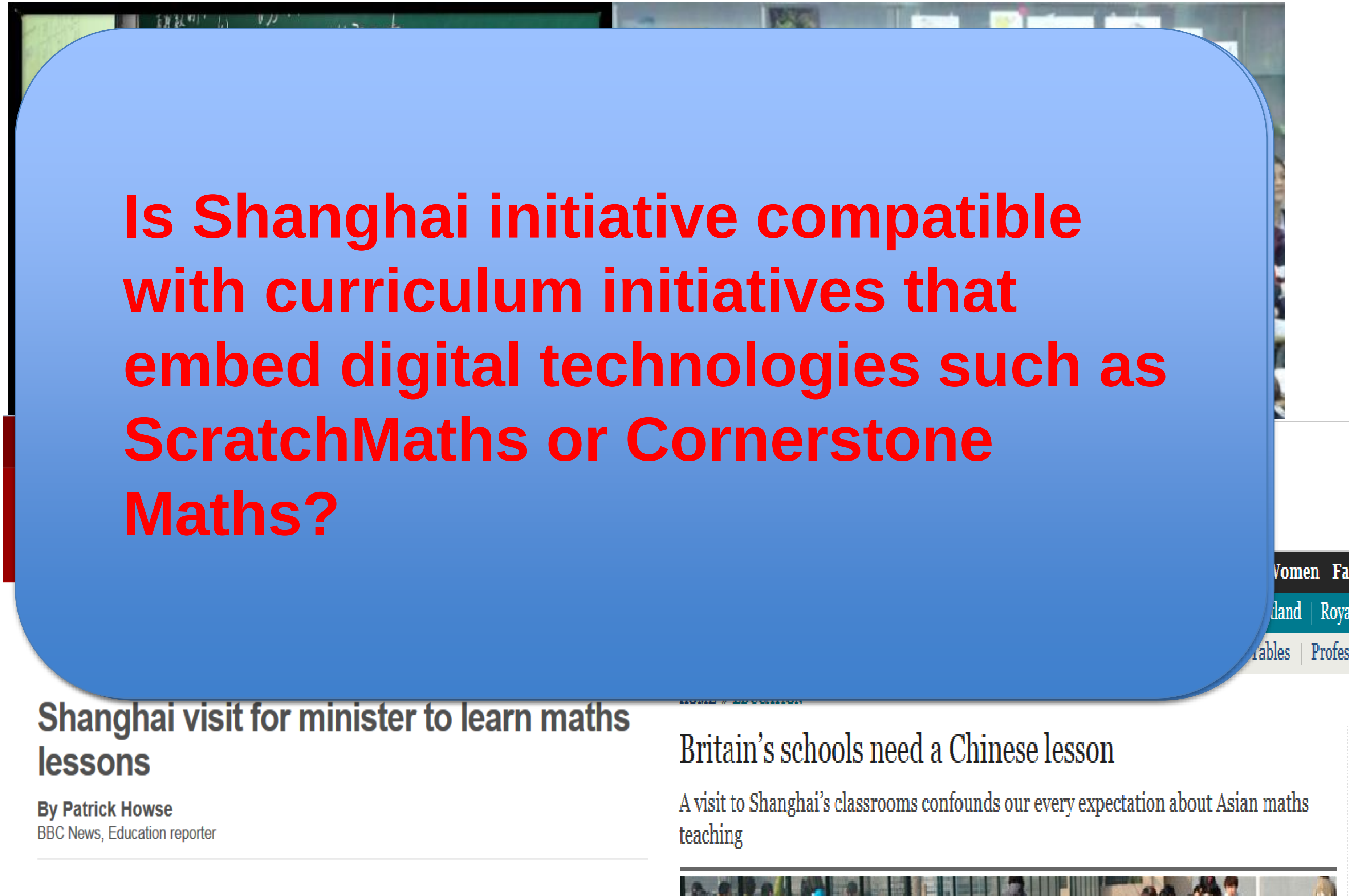
- complexity of the process of integrating digital technology (solid findings in Mathematics Education: European Mathematics Society newsletter March 2014)
- shortage of mathematics teachers, ‘teacher churn’

“If you’re the head of maths you are *so* accountable: if the maths department goes down, the whole school goes down.”

Challenges to SM in England ctd…

- curriculum change
- high stakes mathematics testing
- widespread mathematics anxiety
- Shanghai mathematics initiative

Shanghai initiative



Is Shanghai initiative compatible with curriculum initiatives that embed digital technologies such as ScratchMaths or Cornerstone Maths?

Thank you

谢谢

Questions or comments please